Nonlinear Shrinkage Covariance Matrix Estimation*

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Abstract

Covariance matrix estimates are required in a wide range of applied problems in multivariate data analysis, including portfolio and risk management in finance, factor models and testing in economics, and graphical models and classification in machine learning. In modern applications, where often the model dimensionality is comparable or even larger than the sample size, the classical sample covariance estimator lacks desirable properties, such as consistency, and suffers from eigenvalue spreading. In recent years, improved estimators have been proposed based on the idea of regularization. Specifically, such estimators, known as rotation-equivariant estimators, shrink the sample eigenvalues, while keeping the eigenvectors of the sample covariance estimator. In high dimensions, however, the sample eigenvectors will generally be strongly inconsistent, rendering eigenvalue shrinkage estimators suboptimal. We consider an estimator that goes beyond mere eigenvalue shrinkage and employs recent advancements in random matrix theory to account for eigenvector inconsistency in a large-dimensional setting. We provide the theoretical guarantees and an empirical evaluation demonstrating the superior performance of the proposed estimator.

JEL Classification: C020
Keywords: Shrinkage estimator, Rotation equivariance, Random matrix theory, Large-dimensional asymptotics, Bias correction, Principal components

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